qualitatively many of the features seen in the measured profiles. Foremost is the off-axis pressure minimum. A progressive decrease in pressure in the interval $0.25 \le r/r_0 \le 1.0$ is also noted. The discrepancy in the magnitude of the central pressure drop is partially due to the use of a fixed core radius. It should be mentioned that the positive values of $\Delta p^*/\frac{1}{2}\rho u^2(r_0)$ for small r and θ result from the fact that $\Delta p(0,0)=0$ and that $p(0,0)-p(r_{\mathrm{ref}},0)$ is positive for small r and θ.

Several deficiencies in Sullivan's solution should be noted. Most importantly, there is no coupling between v and the u, w field. This is due to the assumption of the forms of u and w at the outset, and then solving the tangential momentum equation to find a consistent v field, all subject to a free-slip condition on the lower surface. The u, w field has unrealistic far-field conditions since $u(r) \to \infty$ as $r \to \infty$, and $w(r,z) \to \infty$ as $z \to \infty$. The free-slip condition eliminates endwall boundary-layer effects, which are known to be important in swirling flows. As a consequence of this lack of coupling between the tangential and flow-through fields, the model is limited in its ability to properly respond to changes in background conditions (e.g., the radius of maximum tangential velocity is a constant independent of both the height z and the far-field circulation Γ; further, the intensity of the downflow along the certerline is independent of Γ). Also, the viscosity ν is taken to be constant throughout the volume. While this is a good assumption for laminar vortices, recent work by Lewellen and Teske[8] has shown that use of a constant "eddy" viscosity value to model turbulent vortices is not a valid approach.

These considerations limit the usefulness of Eq. (4) in modeling the experimentally observed pressure profiles. While good fits to the profiles in Fig. 1 could be obtained by selecting a different value of ν for each value of Γ (a being held fixed), the physical justification for doing so is not clear. However, in conclusion, it is noted that Sullivan's solution does appear to incorporate some of the essential physics of actual two-celled vortices. In particular, the role of the downdraft along the centerline in creating the central pressure hump is well brought out. Also, the overall progression of the shape of the radial profile of the wall static pressure on the lower surface as the input angular momentum is increased is also qualitatively reproduced.

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Stiffness Matrix Correction from Incomplete Test Data

80006

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Nomenclature

d	= weighted norm of the errors between the given and the optimal stiffness matrix
I	= unity matrix
K	= stiffness matrix
k_{ij}	= ij element of K
M	= mass matrix
m_{ij}	= ij element of M
Y	= optimal stiffness matrix
y_{ij}	= ij element of Y
Z	= general-coordinates vector
Φ	= orthogonal mode shape matrix
Φ_{ij}	= ij element of Φ
ψ	= Lagrange function for stiffness matrix
β_y	= matrix of Lagrange multiplier
$\beta_{y,ij}$	= ij element of β_y
Λ_y	= matrix of Lagrange multiplier
$\lambda_{y,ij}$	= ij element of Λ_y
Ω^2	= measured frequency matrix
ω_{ij}^2	= ij element of Ω^2
$(\ )^T$	= transpose of ( )

Introduction

In Refs. 1 and 2, Baruch and Bar-Itzhack use the Lagrange multiplier method to optimally correct the stiffness matrix from test data. The resulting stiffness matrix which satisfies the dynamic equation is symmetric. However, the Lagrange multiplier Λ_y, which is used to define the Lagrange function ψ, is difficult to obtain without making any assumption. The crucial assumption for the matrix $\Lambda_y^T MX$, which is assumed to be symmetric in Ref. 2, is not always true in general and is hard to understand from a physical point of view. The main purpose of this Technical Note is to propose a new approach which shows the uniqueness of the corrected stiffness matrix in a different way than that given in Ref. 2. The corrected stiffness matrix Y can be directly obtained without making any assumptions. It is very interesting that by algebraic manipulation the Lagrange multiplier Λ_y can be eliminated from the derivation. Thus, the necessity to calculate Λ_y can be avoided.

Optimization Procedure for Correcting the Stiffness Matrix

In an incomplete test, the measured modal matrix $\Phi(n\times m)$ is rectangular, where $n \ge m$, and the mass matrix $M(n\times n)$ is a positive-definite symmetric matrix. The mode shapes and mass matrix must be orthogonal to fulfill the basic requirement before starting to correct the stiffness matrix.

$$\Phi^T M \Phi = I \tag{1}$$

Following Refs. 1 and 2, the simplest way to find a symmetric stiffness matrix Y, which is closest to the measured or computed stiffness matrix K, is given by minimizing the norm

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$$d = \tfrac{1}{2}\| M^{-\frac{1}{2}}(Y-K)M^{-\frac{1}{2}}\| = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}$$

$$\left[\sum_{q=1}^{n} m_{iq}^{-\frac{1}{2}} \sum_{p=1}^{n} (y_{qp}-k_{qp}) m_{pj}^{-\frac{1}{2}}\right]^2 \tag{2}$$

Y is the desired corrected stiffness matrix, and K is a known approximate stiffness matrix obtained either by a finite-element method or by measurements. Here K is symmetric and can be singular if it includes rigid-body motions. Y must satisfy the constraints

$$Y\Phi = M\Phi\Omega^2 \tag{3}$$

$$Y = Y^T \tag{4}$$

where Ω^2 $(m \times m)$ is a diagonal matrix which represents the measured frequencies. The constraints must be necessary and sufficient for Φ and Ω^2 to be solutions of the dynamic equation

$$M\ddot{Z} + YZ = 0 \tag{5}$$

Using Lagrange multipliers to include the constraints of Eqs. (3) and (4), the Lagrange function (ψ) is defined as follows:

$$\psi \equiv d + 2\Pi\Lambda_y(Y\Phi - M\Phi\Omega^2)\Pi + \Pi\beta_y(Y-Y^T)\Pi \tag{6}$$

where

$$\Pi\Lambda_y(Y\Phi - M\Phi\Omega^2)\Pi = \sum_{i=1}^{n}\sum_{j=1}^{m}(\lambda_y)_{ij}\Bigg(\sum_{r=1}^{n} y_{ir}\Phi_{rj}$$

$$- \sum_{r=1}^{n} m_{ir}\sum_{q=1}^{m}\Phi_{rq}\omega_{qj}^2\Bigg) \tag{7a}$$

and

$$\Pi\beta_y(Y-Y^T)\Pi = \sum_{i=1}^{n}\sum_{j=1}^{n}(\beta_y)_{ij}(y_{ij}-y_{ji}) \tag{7b}$$

Here Λ_y is a rectangular matrix of order $(n \times m)$, and β_y is an antisymmetric matrix of order $(n \times n)$.

$$\beta_y = -\beta_y^T \tag{8}$$

and

$$\omega_{pq}^2 = \omega_{pp}^2 \quad \text{for } p=q \tag{9a}$$

$$\omega_{pq}^2 = 0 \quad \text{for } p \neq q \tag{9b}$$

where ω_{pp} are the measured frequencies.

The partial differentiation of ψ with respect to y_{ij} is set to zero, resulting in the y_{ij} for minimal ψ. In matrix form, one can get

$$\left[\frac{\partial\psi}{\partial y_{ij}}\right] = M^{-1}(Y-K)M^{-1} + 2\Lambda_y\Phi^T + 2\beta_y = 0 \tag{10}$$

Adding Eq. (10) and its transpose to eliminate β_y yields

$$Y = K - M\Lambda_y\Phi^T M - M\Phi\Lambda_y^T M \tag{11}$$

Substituting Eq. (11) into Eq. (3) yields

$$M\Phi\Omega^2 = K\Phi - M\Lambda_y - M\Phi\Lambda_y^T M\Phi \tag{12}$$

Rearranging Eq. (12) yields

$$M\Lambda_y = K\Phi - M\Phi\Omega^2 - M\Phi\Lambda_y^T M\Phi \tag{13}$$

Taking the transpose of Eq. (13) yields

$$\Lambda_y^T M = \Phi^T K - \Omega^2\Phi^T M - \Phi^T M\Lambda_y\Phi^T M \tag{14}$$

Substitution of Eqs. (13) and (14) into Eq. (11) becomes

$$Y = K - K\Phi\Phi^T M + 2M\Phi\Omega^2\Phi^T M + M\Phi\Lambda_y^T M\Phi\Phi^T M$$

$$- M\Phi\Phi^T K + M\Phi\Phi^T M\Lambda_y\Phi^T M \tag{15}$$

Again, Eqs. (13) and (14) are substituted into Eq. (15) to get

$$Y = K - K\Phi\Phi^T M - M\Phi\Phi^T K + 2M\Phi\Phi^T K\Phi\Phi^T M$$

$$- M\Phi\Phi^T M\Lambda_y\Phi^T M - M\Phi\Lambda_y{}^T M\Phi\Phi^T M \tag{16}$$

Adding Eqs. (15) and (16) results in the solution of Y.

$$Y = K - K\Phi\Phi^T M - M\Phi\Phi^T K + M\Phi\Omega^2\Phi^T M + M\Phi\Phi^T K\Phi\Phi^T M \tag{17}$$

Conclusions

The stiffness matrix can be optimally corrected by using the Lagrange multiplier method. The corrected stiffness matrix Y can be directly obtained without making any assumptions, and the proof given is unique.

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Influence of Nonlinear Adhesive Behavior on Analysis of Cracked Adhesively Bonded Structures

40004
80005

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Introduction

ADHESIVE bonding is finding increased use in aerospace structures due to cost reduction and improved structural efficiency. The use of adhesive bonding in primary structures

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